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PREPARING FOR QUANTUM: MAPPING TSP AND VRP IN QUBO FORMULATION

2nd Slovenian Forum on Quantum Computing: From Research Achievements to Own Quantum Capabilities

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  - Step by step: Traveling Salesman Problem (TSP) → VRP → Electric VRP (EVRP)



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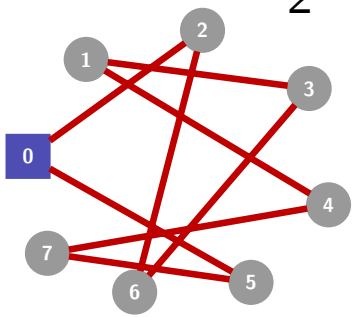
- Background: years of solving vehicle routing problems (VRPs) with classical methods
- Goal: explore the possibilities of quantum computing for VRPs
  - Step by step: Traveling Salesman Problem (TSP) → VRP → Electric VRP (EVRP)
- Quantum computers do not take routing models directly
  - Hardware-compatible form → Quadratic Unconstrained Binary Optimization (QUBO)
  - Other models: HUBO (higher-order terms), CQM (constrained, hybrid solvers) etc.
- Questions
  - How do we map TSP/VRP to QUBO?
  - How does the chosen formulation affect qubit count and solution quality?
  - Are the models ready for today's quantum hardware?

The Traveling Salesman Problem

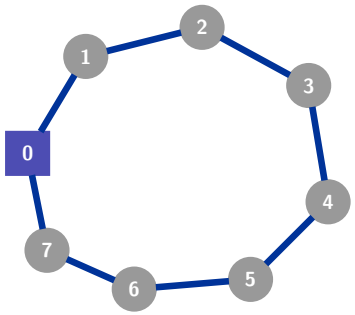
- Find the minimum-distance closed tour that visits every city exactly once and returns to the start

Symmetric TSP ($d_{ij} = d_{ji}$):

$$\frac{(n-1)!}{2} \text{ tours}$$



Non-optimal tour



Optimal tour

Asymmetric TSP ($d_{ij} \neq d_{ji}$):

$(n-1)!$ tours

Cities	Tours (symmetric)
5	12
10	181 440
15	43 589 145 600
20	$6.08 \cdot 10^{16}$

- NP-hard; small instances with known optima $\Rightarrow$ ideal test bed for quantum optimisation

QUBO model

- Quadratic Unconstrained Binary Optimization

$$\min_{x\in\{0,1\}^N} x^\top Q x = \sum_i Q_{ii} x_i + \sum_{i<j} Q_{ij} x_i x_j$$

- Constraints $Ax = b$ become quadratic penalties

$$E(x) = \underbrace{C(x)}_{\text{route cost}} + \lambda \underbrace{\|Ax - b\|^2}_{P(x)}$$

- λ large enough to penalise infeasible states, but not so large that the penalty terms overwhelm the route-cost differences
- Equivalent Ising model $\Rightarrow x_i = (1 - s_i)/2$, $s_i \in \{-1, +1\}$

Upper-triangular Q matrix

	x_1	x_2	x_3	x_4
x_1	Q_{11}	Q_{12}	Q_{13}	Q_{14}
x_2	0	Q_{22}	Q_{23}	Q_{24}
x_3	0	0	Q_{33}	Q_{34}
x_4	0	0	0	Q_{44}

diagonal = linear terms
off-diagonal = interactions

Position-based TSP formulation

- $x_{i,p} = 1$ if city i is visited at position p, $i, p \in \{0, \ldots, n-1\}$

$$\sum_p x_{i,p} = 1 \quad \forall i \quad \text{(each city visited once)}$$

$$\sum_i x_{i,p} = 1 \quad \forall p \quad \text{(each position holds one city)}$$

$$E(x) = \sum_i \sum_{j \neq i} d_{ij} \sum_p x_{i,p}\, x_{j,p+1} + \lambda \sum_i \left(1 - \sum_p x_{i,p}\right)^2 + \lambda \sum_p \left(1 - \sum_i x_{i,p}\right)^2$$

- Variables: n^2
- Encodes the tour order directly $\Rightarrow$ no subtours

Assignment matrix

tour $0 \to 2 \to 1 \to 3 \to 0$

	p=0	p=1	p=2	p=3
i=0	**1**	0	0	0
i=1	0	0	**1**	0
i=2	0	**1**	0	0
i=3	0	0	0	**1**

(values of $x_{i,p}$, not the Q matrix)

Edge-based TSP formulations

Directed: $x_{ij} = 1$ if the tour goes $i \to j$, $i \neq j$

$$\sum_{j \neq i} x_{ij} = 1 \quad \forall i \qquad \text{(one outgoing edge)}$$

$$\sum_{i \neq j} x_{ij} = 1 \quad \forall j \qquad \text{(one incoming edge)}$$

$$E(x) = \sum_i \sum_{j \neq i} d_{ij}\, x_{ij} + \lambda \sum_i \left(1 - \sum_{j \neq i} x_{ij}\right)^2 + \lambda \sum_j \left(1 - \sum_{i \neq j} x_{ij}\right)^2$$

- Variables: $n(n-1)$

Undirected: $x_{ij} = 1$ if edge $\{i, j\}$ is used, $i < j$

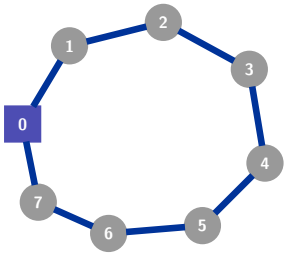
$$\sum_{j<i} x_{ji} + \sum_{j>i} x_{ij} = 2 \quad \forall i$$

$$E(x) = \sum_i \sum_{j>i} d_{ij}\, x_{ij} + \lambda \sum_i \left(2 - \sum_{j<i} x_{ji} - \sum_{j>i} x_{ij}\right)^2$$

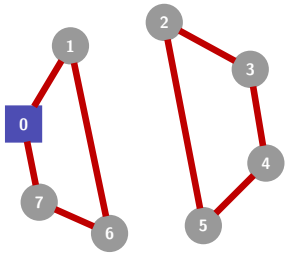
- Two edges connect each city
- Variables: $n(n-1)/2$

Degree constraints do **not** guarantee one connected tour $\Rightarrow$ **subtours** can satisfy all penalties. In the undirected model the smallest subtour needs two triangles $\Rightarrow$ **subtour-free for $n \leq 5$ nodes**.

Subtour elimination



One Hamiltonian tour

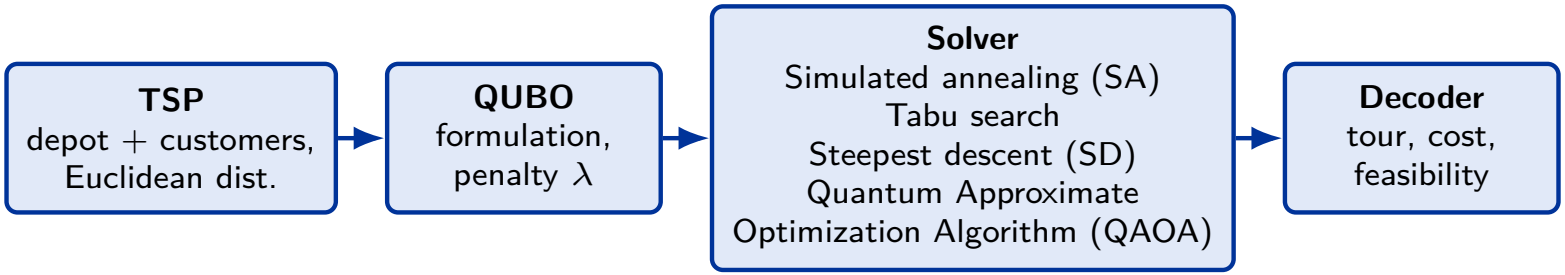


Degrees OK, two cycles

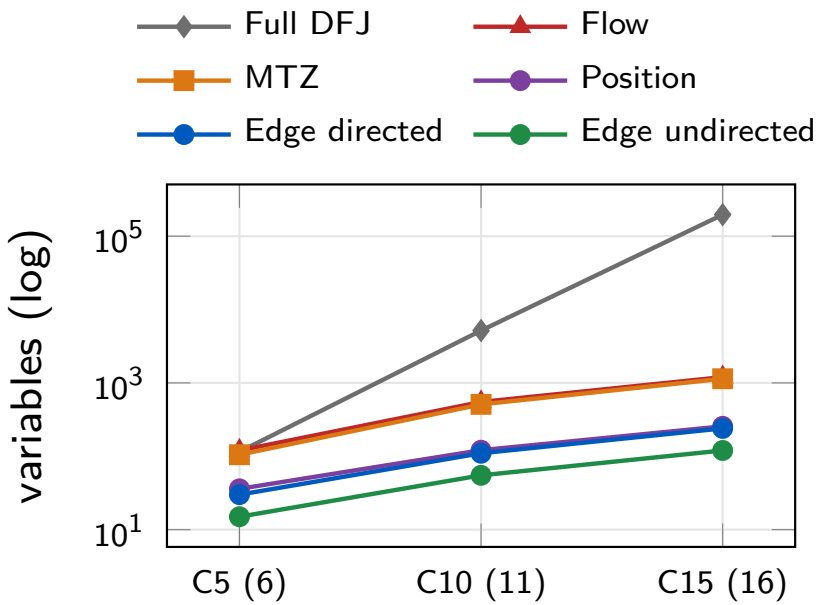
- Miller–Tucker–Zemlin (MTZ): each city gets an order number that must increase along the tour
- Single-commodity flow: the depot sends flow to every city along the used edges
- Dantzig–Fulkerson–Johnson (DFJ): each subset of cities uses fewer edges than it has cities
- Lazy DFJ: DFJ constraints added only for subtours found in the samples

QUBO has no inequalities: inequalities are converted to equalities using **binary slack variables**, then encoded as QUBO penalties $\Rightarrow$ subtour elimination requires extra qubits

Experimental setup and model size



- TSP instances: C5, C10, C15 (5/10/15 customers + depot)
- Six QUBO models: position, edge directed/undirected, directed + MTZ/Flow/DFJ
- SA, 1 000 reads - no quantum hardware
- Subtour elimination increases the model size



Results: feasibility and solution quality

Model (SA, 1 000 reads)	Feasibility rate (%)			Optimality gap (%)		
	C5	C10	C15	C5	C10	C15
Position	100.0	99.0	35.7	0.00	0.00	0.00
Edge undirected (no SEC*)	85.3	57.3	19.7	0.00	2.14	1.11
Edge directed (no SEC*)	39.6	16.6	1.5	0.00	0.00	11.62
MTZ	43.8	0.1	0.0	0.00	41.54	–
Flow	36.9	18.7	2.2	0.00	2.91	11.69
Lazy DFJ	63.2	24.5	3.0	0.00	7.90	24.52

* SEC = subtour elimination constraints; MTZ, Flow and Lazy DFJ are added to the directed edge model

- Position-based: highest feasibility and optimal on every instance ⇒ subtours cannot occur
- Undirected edge: best edge model with half the variables of the directed one
- Subtour elimination adds variables and does not consistently improve feasibility
- Quality drops significantly with instance size

Validation of the QUBO models

- Exhaustive enumeration of all 2^N states for $N \leq 30$, exact penalty threshold $\lambda^* = \max_{x:\, P(x) > P_{\min}} \frac{C_{\text{opt}} - C(x)}{P(x) - P_{\min}}$

n	Formulation	N	Feasible (%)	Subtours (%)	Min = opt.	λ	λ^*
4	Undirected edge	6	4.69	0	yes	50	19.04
4	Depot-fixed position	9	1.17	–	yes	50	34.04
4	Directed edge	12	0.146	0.0732	no	50	23.93†
4	Full position	16	0.0366	–	yes	50	34.04
5	Undirected edge	10	1.17	0	yes	50	15.21
5	Depot-fixed position	16	0.0366	–	yes	50	30.20
5	Directed edge	20	$2.29 \cdot 10^{-3}$	$1.91 \cdot 10^{-3}$	yes	50	20.67
5	Full position	25	$3.58 \cdot 10^{-4}$	–	yes	50	30.20
6	Undirected edge	15	0.183	0.0305	yes	60	18.03
6	Depot-fixed position	25	$3.58 \cdot 10^{-4}$	–	yes	60	32.12
6	Directed edge	30	$1.12 \cdot 10^{-5}$	$1.35 \cdot 10^{-5}$	no	60	18.94†

† over non-subtour states only (a subtour cheaper than the optimal tour exists)

- 9 of 11 cases: global minimum at default λ is the optimal tour ($\lambda = 1.47$–$3.33\,\lambda^*$)
- Directed edge, $n = 4, 6$: a subtour is cheaper than the optimal tour $\Rightarrow$ no λ fixes it

CVaR-QAOA on an ideal simulator

- CVaR-QAOA: $\alpha = 0.2$, COBYLA ($\leq$100 it.), 1 000 shots, Qiskit Aer (noiseless), 3 seeds, 3 layers (p[1])

Case	Qubits	P(feasible), % of shots				P(optimal), % of shots			
		p=1	p=2	p=3	*random*	p=1	p=2	p=3	*random*
n=4 undirected	6	32.0	**46.5**	44.6	*4.69*	10.13	**19.60**	14.33	*1.56*
n=4 depot-fixed	9	7.5	8.1	**12.1**	*1.17*	2.90	3.83	**4.83**	*0.391*
n=4 directed	12	1.8	**4.4**	2.5	*0.146*	0.80	**1.47**	1.13	*0.0488*
n=5 undirected	10	9.5	7.1	**13.4**	*1.17*	2.07	0.70	**10.50**	*0.0977*
n=5 depot-fixed	16	1.2	1.4	**3.4**	*0.0366*	0.10	0.17	**0.43**	*0.00305*
n=6 undirected	15	2.5	**4.5**	1.3	*0.183*	0.00	0.03	0.03	*0.00305*

Mean over 3 seeds. Random = share of feasible / optimal states in the whole state space.

- Feasibility above the random-guessing baseline in 18 of 18 settings
- Optimum found in all seeds for $n = 4$; rare at 15 qubits

[1] p = number of QAOA layers (cost + mixer operator), i.e. $2p$ circuit angles γ, β; CVaR_α = objective that averages the energy of only the best $\alpha \cdot 100$ % of shots ($\alpha = 0.2$: best 200 of 1 000); COBYLA = gradient-free classical optimiser of the angles, at most 100 iterations (circuit evaluations); seed = initial angles and simulator seed.

Circuit size: connectivity matters

- Non-zero coupling = two variables multiplied in the QUBO: from penalties (two variables in the same constraint) and, in the position model, from the cost term $x_{i,p}x_{j,p+1}$
- Each coupling $\Rightarrow$ 2 CZ gates; limited connectivity $\Rightarrow$ extra SWAP gates; $p = 1$ circuits transpiled (Qiskit, opt. level 3, basis $\{cz, rz, sx, x\}$) to all-to-all and a 6×9 grid[1]

				All-to-all			**Grid** 6×9			**Ratio**
n	**Formulation**	**Qubits**	**Couplings**	2q gates	depth	2q depth	2q gates	depth	2q depth	2q gates
4	Undirected	6	12	24	65	16	36	129	34	1.50
4	Depot-fixed	9	30	60	114	30	93	254	70	1.55
4	Directed	12	24	48	106	26	72	183	50	1.50
5	Undirected	10	30	60	107	28	93	185	54	1.55
5	Depot-fixed	16	84	168	212	58	305	604	192	1.82
5	Directed	20	60	120	186	50	222	355	109	1.85

- On the grid: 1.50–1.85$\times$ more two-qubit gates, 1.73–2.85$\times$ deeper circuits; two-qubit gates dominate the error $\Rightarrow$ formulation $\times$ topology must be tested on real hardware

EuroHPC quantum access

EHPC-QCP-2026Q03-093 – *Towards quantum electric vehicle routing: benchmarking TSP QUBO formulations on analogue and digital EuroHPC quantum computers*

PI: D. Davidović (IRB) · 6 months

System	Model, qubit technology	Qubits, connectivity	QPU h	Status
VLQ	Digital, superconducting	24, star	10	**granted**
Euro-Q-Exa	Digital, superconducting	53, lattice	10	**granted**
PIAST-Q	Digital, trapped ions	20, all-to-all	20	**granted**
Lucy	Digital, photonic	10, all-to-all	10	**granted**
Ruby	Analogue, neutral atoms	100, 2D array	25	not granted

Digital = gate-based quantum computer; analogue = Hamiltonian evolution of the atom register

- Phase 1: cross-platform baseline, n=4 undirected edge (6 qubits)
- Phase 2: formulation trade-off, depot-fixed (9 q.), directed edge (12 q.)
- Phase 3: scale to 5–6 nodes where qubits allow
- Compare with ideal/noisy simulators and random guessing

Conclusion

- Mapping routing problems to QUBO is the key step before any quantum run
  - The formulation decides qubit count, coupling density, subtour risk and solution quality
  - The same QUBO formulation yields very different circuits depending on the hardware topology

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- Mapping routing problems to QUBO is the key step before any quantum run
  - The formulation decides qubit count, coupling density, subtour risk and solution quality
  - The same QUBO formulation yields very different circuits depending on the hardware topology
- Readiness: QUBO models validated
- Next steps
  - Test the formulation $\times$ topology combination on real hardware - the goal of the EuroHPC project: effect of noise and connectivity
  - Transfer simulator-optimised QAOA parameters to hardware, error mitigation
  - Extend from TSP to VRP/EVRP:
    - Vehicle index on every variable $\Rightarrow$ about K times more qubits
    - Capacity, time windows and battery are inequalities $\Rightarrow$ slack qubits, unbalanced penalisation or Lagrangian relaxation
    - Problem decomposition: classical clustering/assignment $\Rightarrow$ quantum TSP per route

Questions



Thank you for your attention!

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